ALUMINIUM EXTRUSION ANALYSIS BY THE FINITE VOLUME METHOD

JOSÉ D. BRESSAN[†], MARCELO M. MARTINS[*] AND SÉRGIO T. BUTTON[§]

[†] Department of Mechanical Engineering
Centre for Technological Science - University of Santa Catarina State (UDESC)
Campus Universitário Prof. Avelino Marcante, s/n
89219-710, Joinville - SC, Brazil.
e-mail: dem2jdb@joinville.udesc.br, www.joinville.udesc.br/

[*] Centre University Catholic of Santa Catarina - Joinville
Campus Centro - Visconde de Taunay 427, 89203-005 Joinville - SC, Brazil.
e-mail: marcelo.martins@catolicasc.org.br, www.catolicasc.org.br/

[§] Department of Materials Engineering
Faculty of Mechanical Engineering - State University of Campinas (UNICAMP)
Campus Cidade Universitário "Zeferino Vaz", Mendeleyev 200
13083-860, Campinas - SP, Brazil.
e-mail: sergio1@fem.unicamp.br, www.fem.unicamp.br/

Key words: Finite Volume, Cold Extrusion, Aluminium, Velocity field.

Abstract. Present work proposes a novel numerical scheme to calculate stress and velocity fields of metal flow in axisymmetric extrusion process in steady state. Extrusion of aluminium is one main metal forming process largely applied in manufacturing bars and products with complex cross section shape. The upper-bound, slab, slip-line methods and more recently the numerical methods such as the Finite Element Method have been commonly applied in aluminium extrusion analysis. However, recently in the academy, the Finite Volume Method has been developed for metal flow analysis: literature suggests that extrusion of metals can be modelled by the flow formulation. Hence, metal flow can be mathematically modelled such us an incompressible non linear viscous fluid, owing to volume constancy and varying viscosity in metal forming. The governing equations were discretized by the Finite Volume Method, using the Explicit MacCormack Method in structured and collocated mesh. The MacCormack Method is commonly used to simulate compressible fluid flow by the finite volume method. However, metal plastic flow and incompressible fluid flow do not present state equations for the evolution of pressure, and therefore, a velocity-pressure coupling method is necessary to obtain a consistent velocity and pressure fields. The SIMPLE Method was applied to attain pressure-velocity coupling. This new numerical scheme was applied to forward hot extrusion process of an aluminium alloy. The metal extrusion velocity fields achieved fast convergence and a good agreement with experimental results. The MacCormack Method applied to metal extrusion produced consistent results without the need of artificial viscosity as employed by the compressible flow simulation approaches. Therefore, present numerical results also

suggest that MacCormack method together with SIMPLE method can be applied in the solution of metal forming processes in addition to the traditional application for compressible fluid flow.

1 INTRODUCTION

With the globalization of economy, market dynamics and growing environmental requirements for sustainable development, the industrial competition has increased and forced the reengineering of all activities at factories and academy in order to reduce the production costs, part weight, energy content, efficient resource use and increase material recyclability.

In the actual engineering practice inside the industries there is the formation of work teams to study and improve the technological processes with the aim at advancing the product quality, durability, sustainability and equipment productivity, as well as to reduce production and maintenance costs. In a broad sense, the industries (automotive, aerospace, stamping, steel, etc.) should continue to attain these objectives in addition to introduce High Strength Low Alloy steels - HSLAS, Advanced High Strength steels - AHSS, to increase the use of lightweight aluminium alloys and magnesium alloys, to produce precision parts with zero-defects, to advance the use of near net shape processes and design for recycling.

Consequently, nowadays in engineering practice, industrial fabrication processes are being investigated and modelled increasingly by the use of software and computers. Numerical and analytical modelling of fabrication processes has great potential to increase process velocity and quality, as well as to reduce costs, through the following factors:

- reduction of the iterations numbers in the experimental method of try-and-error,
- allows a quick construction of modelling or prototype,
- provides a knowledge of the essential physical mechanisms necessary to control the process in real time,
- improves the visualization of the process.

These factors have given recently support and momentum to research and development of mathematical techniques, experimental methods and software for numerical simulations of industrial processes. The main features of the modelling activities carried out in research and development of industrial processes will be summarized briefly below.

In addition to the numerical technique to solve the basic equations, fabrication processes involves some combinations of the following types of material behaviour:

- fluid flow (metal casting, polymer injection, etc.),
- heat transfer (solidification of molten metal, hot forming of metals, cutting, hot pressing and sintering of metal powders, heat treatment of steel),
- plastic deformations (metal forming, metal cutting),
- microstructural and properties evolution (metal casting, welding, cold and hot forming, heat treatment).

Therefore, mathematical modelling of thermo-mechanical process requires an appropriate mathematical formulation to the following conditions:

a- material behaviour during the focused process (elastic and plastic straining, fluid flow and heat transfer),

b- appropriate boundary conditions to the problem (stress, strain and velocity at free or contact surface, friction between piece-die interface or between chip and tool, etc.).

Analytical modelling or numerical simulations allows the calculations of field variables such as the equivalent yield stress and strain, strain and stress components, strain rate, velocity, temperature, from which one can predict the following outcomes:

- geometrical distortions of product and die, residual stresses,
- microstructural parameters to predict yield stress, plastic anisotropy, toughness, etc., such as: grain size and orientation, precipitate state,
- defects in micro-structure: porosity, cracks, surface finish.

Summarising, the mathematical techniques and the experimental methods available for solving fabrication process modelling can be classified into the followings approaches:

- *Material simulation (plasticine, lead, aluminium: stripe pattern grid technique) ,*
- *Physical simulation of process (simple tension test, plastic torsion test, Gleeb test) ,*
- *Analytical methods (slab method, upper bound method, slip line field method) ,*
- *Numerical simulations with mesh (finite element, finite difference, finite volume).*

1.1 Brief History of Metal Extrusion Analysis

In the metal forming industry, by forging, rolling, extrusion and drawing, various kinds of important metal products are fabricated. These processes, axi-symmetrical and plane strain deformations have been investigated using the upper-bound, slab, slip-line and more recently the numerical methods due to increase in computational power [1, 2, 3, 4]. The outcomes of these mathematical modelling analyses have been applied to practice with success.

In hot extrusion processes, the knowledge of metal flow behavior through die and variables such as the forces acting on tool, flow lines, velocity field, deformations, stresses, friction and temperature can help to predict failure and geometrical distortions in components or products produced. One approach to determine such information are employing numerical methods, that are fast and accurate and by this manner, helping to reduce the development time and cost of tooling and optimization operation conditions that could appear in the experimental tryout process. There are three principal defects to be avoided in metal extrusion: surface cracking, extrusion defect, and internal cracking or internal burst.

Numerous materials can be extruded to a wide variety of cross-sectional shapes and dimensions. Considerable research effort in metal forming of products with complex cross section such as plane bars, circular cross section bars, hollow bars, T and H-shape cross section bars have been also carried out experimentally and theoretically due to its relevance to tool design, understanding of metal flow and to obtain information on geometric changes, imposed straining, stresses, friction and load distributions. To achieve these process insights, material simulation by plasticine, visioplastic strain analysis by split billets, stripe pattern grid technique and numerical simulations have been employed extensively [2, 3, 4, 5, 6].

Direct cold extrusion process of aluminum alloy was studied experimentally in 1955 by Thomsen and Frisch [7], using the classical “scratched grid pattern technique” in split billets to obtain velocity, strain fields and flow lines. However, it was very difficult to visualize the flat die corner region with large deformation and shear zones or at the upper layers where the grid lines have tendency to get erased and, thus, some errors were done in the analysis of the experimental results. The literature on the mechanics of extrusion process by *slab method and slip line field method* is large and up to 1960 have been summarized in the book by Johnson and Kudo [8]. Lately, in the eighties, it can be seen in the books by Johnson et al. [3, 4].

In metal forming process analysis, two different classical mathematical approaches can be employed: the *flow approach* and the *solid approach* [9]. In the flow approach formulation, an Eulerian reference coordinate system with fixed mesh in space is employed, the discretized elements are fixed in space and not in the material, and the material is considered non-linear viscous fluid or visco-plastic incompressible fluid. Thus, the numerical grid is constant and keeps its initial geometrical features during numerical simulation. The flow formulation is effective to calculate material internal variables but is not for moving boundaries [9].

In the solid formulation, a Lagrangian reference coordinate system attached to the moving workpiece or material is used: the discretized elements remain attached to the workpiece and deforms with it. Hence, mesh moves and distorts with material deformation or plastic flow. Material is considered elastic-plastic or viscoplastic solid [9]. The solid approach is very popular and has been effective for large plastic deformation analysis of metal forming by the finite element method. However, has the drawback of occurrence of mesh severe distortion at some critical points which causes error in the calculations of field variables. The solution for this problem is to restart the analysis and use a remeshing technique to increase the accuracy, but with higher computation time [10].

In 1968, Altan and Kobayashi [11] presented a numerical technique for temperature distribution in metal extrusion, using the finite difference method and un-coupled thermo-plastic approach. In the eighties, Sellars at al. [12, 13] proposed a finite difference method for temperature distribution in metal rolling, employing also un-coupled thermo-plastic approach with metallurgical changes during the process.

However, a first attempts to employ the flow formulation with coupled thermo-plastic approach in metal forming were the works by Jain [14] and Dawson and Thompson [15] in the seventies. Also in this decade, the finite element method applied to metal forming was first introduced by Zienkiewicz in 1974 [16, 17]. Although shown to be appropriate, mesh distortion and numerical oscillation occurred in the computed solution which could be avoided by mesh refinement but with expensive computer time, as proposed by Zienkiewicz [10]. FEM is nowadays very popular and widely employed to metal forming process analysis by commercial codes such as ABAQUS and DEFORM.

On the other hand, the finite volume method, FVM, has been developed since the seventies and applied mainly to computational fluid dynamic, CFD. However, in the nineties, there was an increase of interests to use FVM in solid mechanics, initially for problems involving fluid-structure interactions. In recent years, the FVM has been proposed for solid structure elastic stress analysis to calculate stress, strain and displacements [18, 19, 20, 21, 22]. Bressan at al. [23] have also presented a numerical approach for calculating transient thermal stress due to a plate thermal shock, using the FVM.

In the nineties at Imperial College of London, a free code package named OpenFoam was developed by Jasak's [24, 25] for computational fluid dynamic, using the FVM. However, nowadays it is also being used to solve solids mechanic and metal forming problems [26, 27].

The aim of present work is to propose a new numerical scheme to calculate stress and velocity fields of metal flow in the axisymmetric extrusion process in steady state. The governing equations were discretized by the Finite Volume Method, using the Explicit MacCormack Method in structured and collocated mesh [28]. Metal plastic flow does not present state equations for the evolution of pressure, therefore, a velocity-pressure coupling method was necessary to obtain a consistent velocity and pressure fields. The SIMPLE

Method [28] was applied to attain pressure-velocity coupling. Present numerical FVM method was applied to direct hot extrusion through a conical die of an aluminium alloy and compared with experimental results, employing the stripe pattern grid technique [6].

2 MATHEMATICAL MODELLING BY FVM

2.1 Governing Equations

The mathematical modelling of metal plastic flow in direct extrusion process will be treated by FVM, using the flow formulation with a fixed Eulerian reference coordinate system: metal plastic flow will be considered similar to a non-linear viscous fluid flow. Tannehill et al. [28] suggests that viscous fluid flow can be numerically simulated by FVM, satisfying the mass, momentum and energy conservation equations and boundary conditions.

According to the work presented by Martins et al. [29], the governing conservation equations in cylindrical coordinates system (r,θ,z) for axisymmetric case, can be written in compact matrix structure in the following form:

$$\frac{\partial \boldsymbol{Q}}{\partial t}+\frac{1}{r}\frac{\partial (r\boldsymbol{F}_r)}{\partial r}+\frac{\partial \boldsymbol{F}_z}{\partial z}=\boldsymbol{S} \qquad (1)$$

where, r is radial coordinate, z is axial coordinate, $\boldsymbol{Q}$, $\boldsymbol{F_r}$, $\boldsymbol{F}_z$ e $\boldsymbol{S}$ are flux vectors that assume the following format:

$$\boldsymbol{Q}=\begin{Bmatrix}\rho\\ \rho v_r\\ \rho v_z\\ \rho cT\end{Bmatrix} \quad \boldsymbol{F}_r=\begin{Bmatrix}\rho v_r\\ \rho v_r^2-\sigma_{rr}\\ \rho v_r v_z-\sigma_{rz}\\ \rho cTv_r+\dot{q}_r\end{Bmatrix} \quad \boldsymbol{F}_z=\begin{Bmatrix}\rho v_z\\ \rho v_z v_r-\sigma_{rz}\\ \rho v_z^2-\sigma_{zz}\\ \rho cTv_z+\dot{q}_z\end{Bmatrix} \quad \boldsymbol{S}=\begin{Bmatrix}0\\ -\dfrac{\sigma_{\theta\theta}}{r}\\ 0\\ \bar{\sigma}.\dot{\bar{\varepsilon}}\end{Bmatrix} \qquad (2)$$

where ρ is density, σ_{rr}, $\sigma_{\theta\theta}$ and σ_{zz} normal components and σ_{rz} is shear component of Cauchy stress tensor $\boldsymbol{\sigma}$, T is temperature, c is specific heat, $\dot{q}_r$ and $\dot{q}_z$ are components of flux heat vector $\dot{\boldsymbol{q}}$, v_r and v_z are velocity vector components, $\bar{\sigma}$ is the effective stress and $\dot{\bar{\varepsilon}}$ is the effective strain rate, t is time.

2.2 Plasticity Constitutive Equations

Present metal plasticity constitutive equation for axisymmetric plastic flow consider material to be incompressible, isotropic and rigid-viscous-plastic. The relation between the stress tensor and strain rate tensor can be written in the following form:

$$\boldsymbol{\sigma}=-\sigma_m+2\eta\dot{\varepsilon} \qquad (3)$$

where $\boldsymbol{\sigma}$ is the Cauchy stress tensor, $\dot{\varepsilon}$ is the strain rate tensor, σ_m is hydrostatic pressure and η is the metal equivalent viscosity. Hydrostatic pressure inside the solid is calculated by $\sigma_m=(\sigma_{rr}+\sigma_{\theta\theta}+\sigma_{zz})/3$, where σ_{rr} , $\sigma_{\theta\theta}$ and σ_{zz} are principal stresses of the stress tensor. Metal flow equivalent viscosity η is calculated from the *Associated Plastic Flow Rule* [29]:

$$\eta=\frac{1}{3}\frac{\bar{\sigma}}{\dot{\bar{\varepsilon}}} \qquad (4)$$

the effective strain rate $\dot{\bar{\varepsilon}}$ must not assume values less than 10^{-3} to avoid numerical instability.

2.3 Finite Volume Method

Integrating Eq. (1) over the control volume given by Fig. 1a, employing Gauss' Theorem, results in the following equation:

$$\frac{\partial \boldsymbol{Q}_{mn}}{\partial t} = -\frac{1}{V_{mn}}\left\{\frac{1}{r}\left[(r\boldsymbol{F}_r\cdot\boldsymbol{s})_{m,n-\frac{1}{2}} + (r\boldsymbol{F}_r\cdot\boldsymbol{s})_{m,n+\frac{1}{2}}\right] + \left[(\boldsymbol{F}_z\cdot\boldsymbol{s})_{m-\frac{1}{2},n} + (\boldsymbol{F}_z\cdot\boldsymbol{s})_{m+\frac{1}{2},n}\right]\right\} + \boldsymbol{S}_{mn} \tag{5}$$

where $\boldsymbol{s}$ is the outward surface vector and V_{mn} is control volume area.

Explicit MacCormack scheme was applied in the Eq. (5). This scheme use explicit Euler scheme on time. Considering outward surface vector correction on the control volume given by Fig. 1b, thus, Eq. (6) represents the predictor step, Eq. (7) represents the corrector step and Eq. (8) represents the current step. MacCormack scheme is a pseudo-transient process, where Δt is a virtual time increment to obtain the final converged solution.

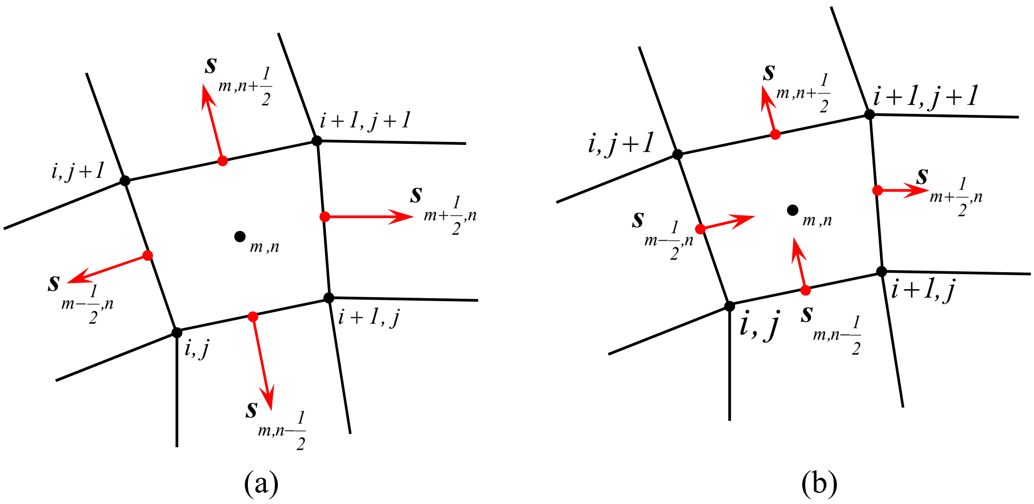

Figure 1. Representation of quadrilateral control volume: (a) with outward vector. (b) outward vector correction.

$$\underline{\boldsymbol{Q}_{mn}^{t+1}} = \boldsymbol{Q}_{mn}^{t} - \frac{\Delta t}{V_{mn}}\left\{\frac{1}{r}\left[\left(r\boldsymbol{F}_{m,n+1}\cdot\boldsymbol{s}_{m,n+\frac{1}{2}}\right) - \left(r\boldsymbol{F}_{m,n}\cdot\boldsymbol{s}_{m,n-\frac{1}{2}}\right)\right] + \left[\left(\boldsymbol{F}_{m+1,n}\cdot\boldsymbol{s}_{m+\frac{1}{2},n}\right) - \left(\boldsymbol{F}_{m,n}\cdot\boldsymbol{s}_{m-\frac{1}{2},n}\right)\right]\right\}^{t} + \Delta t\,\boldsymbol{S}_{mn}^{t} \tag{6}$$

$$\underline{\underline{\boldsymbol{Q}_{mn}^{t+1}}} = \boldsymbol{Q}_{mn}^{t} - \frac{\Delta t}{V_{mn}}\left\{\frac{1}{r}\left[\left(r\underline{\boldsymbol{F}_{m,n}}\cdot\boldsymbol{s}_{m,n+\frac{1}{2}}\right) - \left(r\underline{\boldsymbol{F}_{m,n-1}}\cdot\boldsymbol{s}_{m,n-\frac{1}{2}}\right)\right] + \left[\left(\underline{\boldsymbol{F}_{m,n}}\cdot\boldsymbol{s}_{m+\frac{1}{2},n}\right) - \left(\underline{\boldsymbol{F}_{m-1,n}}\cdot\boldsymbol{s}_{m-\frac{1}{2},n}\right)\right]\right\}^{t} + \Delta t\,\boldsymbol{S}_{mn}^{t} \tag{7}$$

$$\boldsymbol{Q}_{mn}^{t+1} = \frac{1}{2}\left(\underline{\boldsymbol{Q}_{mn}^{t+1}} + \underline{\underline{\boldsymbol{Q}_{mn}^{t+1}}}\right) \tag{8}$$

where, t is current time, $t+1$ is next time step, Δt is a virtual time step, $\underline{Q_{mn}^{t+1}}$ is predictor step, $\underline{\underline{Q_{mn}^{t+1}}}$ is corrector step and Q_{mn}^{t+1} is current step. The flux vector $\boldsymbol{F}$, inside Eq. (6) and Eq. (7), must be discretized in correct way to ensure the main feature of MacCormack method: numerical method of second order accuracy in time and space [28].

The SIMPLE Method was applied to obtain coupling of velocity-pressure variables in the numerical scheme developed in this work [29]. The SIMPLE Method carries out the coupling by corrections in both variables, i.e., velocity and pressure.

The boundary conditions were imposed over the control volume external faces, using the scheme called *"Ghost Volume"*. In these ghost volumes it can be applied boundary condition of *"Dirichlet Kind"* and *"Neumann kind"*. According to Jasak [25] the boundary conditions for physical consistency applied for metal flow in direct extrusion analysis should be such as:

- **Inlet:** the velocity field should be prescribed and pressure should have zero gradient.
- **Outlet:** the velocity should have zero gradient and pressure should be prescribed.
- **Symmetry line:** should be prescribed zero gradients for normal surface gradient and parallel surface components should use domain values.
- **Solid Wall:** at solid wall interface between material and die, consider friction model.

The friction model used was Friction Factor represented by $\tau_{nt} = mk$, where τ_{nt} is the tangential friction stress, k is yield shear stress and *m* is friction factor. Considering isotropic material, von Mises yield criteria and shear strain rate $\dot{\gamma}_{nt}$, the shear friction stress is given by:

$$\tau_{nt} = \frac{2\bar{\sigma}}{3\dot{\bar{\varepsilon}}}\dot{\gamma}_{nt} \tag{9}$$

3 EXPERIMENTAL PROCEDURE

Experimental results of aluminum direct hot extrusion were obtained by the technique known as “stripe pattern grid technique” introduced by Valberg [6]. Contrast pins were inserted axially and radially in the longitudinal symmetry plane of the aluminum billet as seen in Fig. 3. This method presented some experimental difficulties related to the manufacturing of the split die, necessary to remove the extruded billet without damaging it, and to prepare the surfaces to be analyzed, since the billet has to be cut off along its longitudinal section and then polished and etched to reveal the flow pattern. In addition, if the flow is to be evaluated at various extrusion stages, many billets have to be prepared, deformed and analyzed.

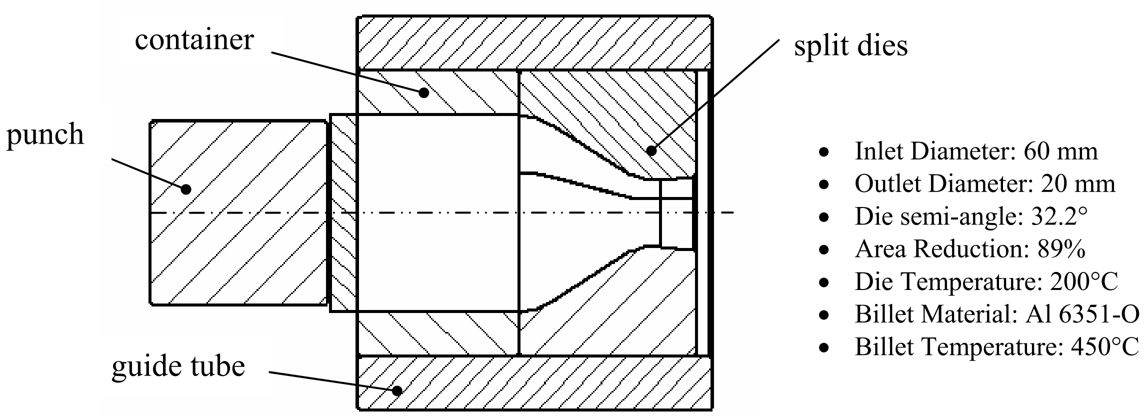

Figure 2. Sketch of experimental extrusion tooling and conditions used in present work.

In Fig. 2, the tooling set up employed in the experimental tests is shown. The extrusion die have the following dimensions: inlet diameter of 60 mm; outlet diameter of 20 mm; work angle of 32.3^o corresponding to an area reduction of 89%.

Billets with 58 mm in diameter and 40 mm long were cut off from an aluminum 6351 T5 bar and machined to receive the contrast pins with 5 mm in diameter made of aluminum 2011: three longitudinal and four radial pins, as shown in Fig. 3. After machining the billet, the workpieces were solubilized at 515 ^oC for 60 minutes and air cooled. Strain ε and strain rate $\dot{\varepsilon}$ hardening behaviour of material was obtained by Hansel-Spittel law [30,31], Eq. (10), from tensile tests at temperature T= 450^{o}C. The fitted parameters of Eq. (10) are shown in Table 1.

$$\sigma_i = A e^{m_1 T} T^{m_9} \varepsilon_i^{m_2} e^{(m_4/\varepsilon_i)} \left(1+\varepsilon_i\right)^{m_5 T} e^{m_7 \varepsilon_i} \dot{\varepsilon}_i^{m_3} \dot{\varepsilon}_i^{m_6 T} \tag{10}$$

Extrusion tests were carried out in a hydraulic press with ram speed of 10 mm/s. Before the tests, the dies were lubricated with a mixture with mineral oil, graphite and molybdenum disulphide. The workpieces were heated at 450 ^oC for 30 minutes and the tools were heated at 200 ^oC to minimize the heat transfer and workpiece cooling.

Extruded workpieces were removed from the container, air cooled and cut off at the medium longitudinal plane, then milled, polished and etched with a reagent (solution of 85 ml H_20, 15 ml HF and 15 ml HNO_3) to reveal the patterns shown in Fig. 3.

Table 1 – Hansel-Spittel law parameters obtained to model the flow stress of aluminum alloys.

Material	A	m_1	m_2	m_3	m_4
AA 2011 (pin)	667.98713	-0.00485	-0.03347	0.08079	-0.00228
AA 6351 (matrix)	953.65542	-0.00524	-0.01407	0.10998	-0.00913

4 RESULTS AND DISCUSSION

Metal flow pattern at longitudinal middle section of one extruded workpiece by the stripe pattern grid technique is shown in Fig. 3. The velocity isolines of metal flow in direct hot extrusion were calculated from this stripe pattern geometry similarly to the classical technique of split billet before extrusion: the initial contrast pin had constant homogeneous diameter and were equally spaced inside the middle longitudinal section of billet. After the extrusion deformation process, the pins were deformed following the metal flow behaviour.

On the other hand, the numerical simulation by the present FVM method was performed employing a mesh of 1110 volumes seen in Fig. 4 together with the material properties and process conditions presented in Table 2. In the numerical code, axisymmetric direct extrusion process, friction factor model, rigid-perfect plastic material and steady state were used in the modelling. The friction factor considered at die wall was m = 0.5 and zero friction was considered at the container wall. Time step was 10^{-15} and total iterations was about 50,000.

Figure 6 shows the calculated experimental and the numerical simulation results of axial velocities contours (isolines) of constant values. The comparisons are in reasonable agreement. In Fig. 7, the obtained radial velocity isolines from present computational code for direct extrusion of aluminum are presented: the values and distribution are as expected.

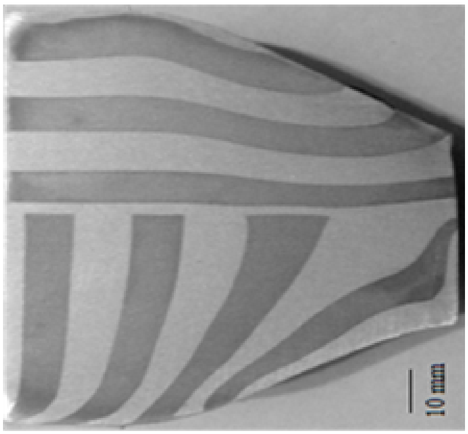

Figure 3. Stripe pattern grid at longitudinal section of extruded aluminum billet.

Tabela 2 - Parameters of material, extrusion process and simulation used in extrusion of aluminum 6351.

Parameters	**Values**
Density (ρ)	2710 (kg.m$^{-3)}$
Yield stress (σ_Y)	255 (MPa)
Area reduction	89 %
Inlet extrusion velocity (V_o)	10 (mm.s^{-1})
Quantity of control volumes	1110
Time step (Δt)	10^{-15} (s)
Die semi-angle (θ)	32.3°
Friction factor parameter (m)	0.5
Material model: rigid-perfectly-plastic	

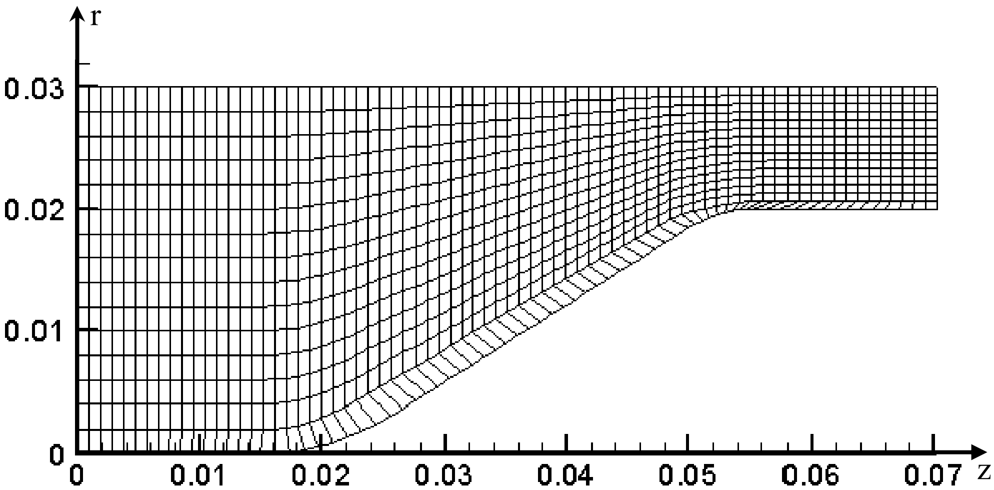

Figure 4. Computational mesh employed in the numerical simulation of Al 6351.

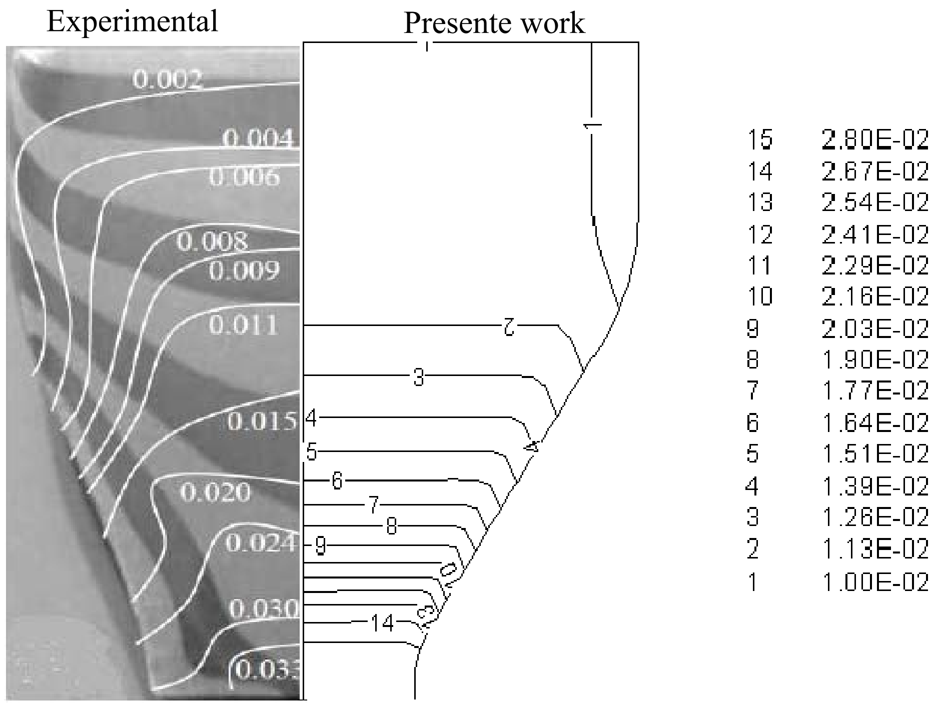

Figura 6. Comparison between the experimental isolines of constant axial velocities v_z (m/s) and the isolines of axial velocities (m/s) from present numerical simulation code for direct extrusion of aluminum.

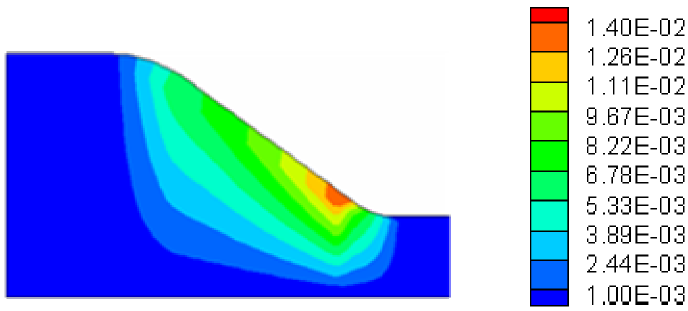

Figure 7. Radial velocity v_r isolines obtained from present computational code for direct extrusion of aluminum.

5 CONCLUSIONS

From the experimental results of constant velocity contours obtained by strip pattern grid technique of direct hot extrusion of aluminum billet and the numerical results from present FVM approach, the following conclusions can be drawn,

- Present numerical scheme is explicit, hence, it is a conditionally stable method. In order to satisfies the CFL condition it was necessary to use a minimum step time of 10^{-15}. This is an extremely low value and, thus, the convergence time was influenced.
- It was necessary about 50,000 iterations to attain numerical convergence.
- Present FVM approach for metal flow did not employed the artificial viscosity for convergence and stability as required by incompressible viscous fluid.

- Present FVM approach together with metal flow formulation has produced good and encouraging results for metal flow in direct extrusion process.
- MacCormack method can also be extended for modeling and analysis of metal flow, as suggested by present work, besides the classical application to incompressible fluid flow analysis.

ACKNOWLEDGMENTS

The authors would like to gratefully thank the financial support given by CNPq of Brazil, University of Santa Catarina State - UDESC, Brazil, UNICAMP/SP/Brazil and CAPES.

REFERENCES

[1] Hill, R., *The Mathematical Theory of Plasticity*. Oxford University Press, (1950).
[2] Avitzur, B, *Metal Forming, the Application of Limit Analysis*. Marcel Dekker, N.Y. (1980)
[3] Johnson,W.; Sowerby, R. and Venter, R.D., *Plaine Strain Slip-Line Fields for Metal Deformation Processes*. Pergamon Press, Oxford, (1982).
[4] Mellor, P.B. and Johnson, W., *Engineering Plasticity*. Van Nostrand Reinhold, (1985).
[5] Kobayashi, S.; OH, S. and Altan, T., *Metal Forming and the Finite-Element Method*. Oxford University Press, (1989).
[6] Valberg, H.S., *Applied Metal Forming: inclunding FEM analysis* edited by Cambridge University Press, London, UK (2010).
[7] Thomsen, E.G. and Frisch, J., Stresses and Strains in Cold-Extruding 2S-O Aluminum, *Trans. ASME.* **77**, (1955) 1343-1353.
[8] Johnson, W. and Kudo, H., *The Mechanics of Metal Extrusion,* edited by Manchester University Press, Manchester, UK (1962).
[9] Ghosh, S., A new finite element description for simulation of metal forming processes. In: E.G. Thompson, R.D. Wood, O.C. Zienkiewicz and A. Samuelson (Eds), *Numerical Methods in Industrial Forming Processes- Numiform 89*, A.A. Balkema, (1989) 159-164.
[10] Zienkiewicz, O.C. and Huang, G.C., Adaptive modelling of transient coupled metal forming processes. In: E.G. Thompson, R.D. Wood, O.C. Zienkiewicz and A. Samuelson (Eds), *Numerical Methods in Industrial Forming Processes- Numiform 89*, A.A. Balkema, (1989) 3-10.
[11] Altan, T. and Kobayashi, S., A numerical method for estimating the temperature distribution in extrusion through conical die. *J. Eng. Ind.*, 90 (1968) 107-118.
[12] Sellars, C.M., The physical metallurgy of hot working. In: C.M. Sellars and G.J. Davies (Eds), *Hot working and forming processes*, Metals Society, London, (1980) 3-15.
[13] Sellars, C.M., Computer modelling of hot-working processes. *Mat. Sci. Eng.*, 1 (1985) 325 – 332.
[14] Jain, P.C., *Plastic flow in solids* (static, quasistatic and dynamic situations including temperature effects), University College of Swansea, Wales, (1976), Ph.D. thesis.
[15] Dawson, P.R. and Thompson, E.G., Steady-state thermomechanic finite element analysis of elastoviscoplastic metal forming processes. *Numerical modelling of manufacturing processes*, ASME, PVP-PB-025, (1977) 167-182.
[16] Zienkiewicz, O.C. and Godbole, P.N., Flow of plastic and viscoplastic solids with special

reference to extrusion and forming processes. *Int. J. Num. Meth. Engng.*, 8 (1974) 3-16.
[17] Zienkiewicz, O.C., Jain, P.C. and Onate, E., Flow of solids during forming and extrusion: some aspects of numerical solutions. *Int. J. Solids Struct.*, 14 (1978) 15-38.
[18] Oñate, E. , Cervera, M. and Zienkiewicz, O.C., A Finite-Volume Format for Structural Mechanics. *Int. J. Num. Meth. Engng.*, 37 (1994) 181–201.
[19] Demirdzic, I. and Martinovic, D., Finite volume method for thermo-elasto-plastic stress analysis. *Computer Meth. Applied Mech. and Engng.*, 109 (1993) 331-349.
[20] Bailey, C. and Cross, M., A finite volume procedure to solve elastic solid mechanics problems in three dimensions on an unstructured mesh. *Int. J. Num. Meth. Engng.*, 38 (1995) 1757–1776.
[21] Greenshields, C.J., Weller, H.G., Ivankovic, A., The finite volume method for coupled fluid flow and stress analysis. *Comput. Model Simul. Eng.,*4 (1999) 213–218.
[22] Wenke, P. and Whell, M.A., A finite volume method for solid mechanics incorporating rotational degrees of freedom. *Computers and Structures*, 81(2003) 321–329.
[23] Bressan, J.D., Martins, M.M. and Vaz Jr., M., Stress evolution and thermal shock computation using the finite volume method. *Journal of Thermal Stresses*, 33 (2010) 533 - 558.
[24] OPENFOAM – The Open Source CFD Toolbox, "Programmer's Guide; Versão 1.6," http://www.openfoam.com/docs/.
[25] Jasak, H., *Error analysis and estimation for the Finite Volume Method with applications to fluid flows*, Ph.D. Thesis, Imperial College London, 1996.
[26] Basic, H.; Demirdzic, I. and Muzaferija, S., Finite volume method for simulation of extrusion processes. *Int. J. Num. Meth. Engng.*, 62 (2005) 475 – 494.
[27] Martins, M.M.; Bressan, J.D.; Button, S.T.; Ivankovic, A., Extrusion process by finite volume method using OpenFoam software. In: F. Chinesta, Y. Chastel, M. El Mansori, (Eds), *International Conference on Advances in Material and Processing Technologies - AMPT2010*, Paris, American Institute of Physics, (2010).
[28] Tannehill, J.C.; Anderson, D.A. and Pletcher, R.H., *Computational Fluid Mechanics and Heat Transfer*. Taylor&Francis, London, (1997).
[29] Martins, M.M.; Bressan, J.D.; Button, S.T., Metal extrusion analysis by finite volume method. In: E. Oñate, D.R.J. Owen, D. Peric and B. Suárez (Eds), *XII International Conference on Computational Plasticity. Fundamentals and Applications. COMPLAS 2013*, Barcelona, (2013).
[30] Hensel, A. and Spittel, T., Kraft- und Arbeitsbedarf bildsamer Formgebungsverfahren. *VEBDeutscher Verlag fur Grundstoffindustrie*, Leipzig, 1978.
[31] Martínez, H.V.; Coupard, D. and Girot, F., *J. Mat. Proc. Techn.*,173 (2006) 252-259.